# The Legacy of Modern Portfolio Theory

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In 1952 *The Journal of Finance* published an article titled "Portfolio Selection" authored by Harry Markowitz. The ideas introduced in this article have come to form the foundations of what is now popularly referred to as Modern Portfolio Theory (MPT). Initially, MPT generated relatively little interest, but with time, the financial community strongly adopted the thesis, and now 50 years later, financial models based on those very same principles are constantly being reinvented to incorporate all the new findings that result from that seminal work.

An important outcome of the research generated due to the ideas formalized in MPT is that today's investment professionals and investors are very different from those 50 years ago. Not only are they more financially sophisticated, but they are armed with many more tools and concepts. This allows both investment professionals to better serve the needs of their clients, and investors to monitor and evaluate the performance of their investments.[1]

Though widely applicable, MPT has had the most influence in the practice of portfolio management. In its simplest form, MPT provides a framework to construct and select portfolios based on the expected performance of the investments and the risk appetite of the investor. MPT, also commonly referred to as mean-variance analysis, introduced a whole new terminology which now has become the norm in the area of investment management. Therefore, on the 50th anniversary of the birth of MPT it may be instructive for investment professionals to revisit the building blocks of the profession. This will serve as an overview of the theory and may enable us to appreciate some of the finer nuances of MPT which we now take for granted. In the process we will also gain an understanding of the advent of modern portfolio management, i.e., where it came from and where it is going.

It may be useful to mention here that the theory of portfolio selection is a normative theory. A normative theory is one that describes a standard or norm of behavior that investors should pursue in constructing a portfolio, in contrast to a theory that is actually followed. Asset pricing theory such as the capital asset pricing model goes on to formalize the relationship that should exist between asset returns and risk if investors constructed and selected portfolios according to mean-variance analysis. In contrast to a normative theory, asset pricing theory is a positive theory—a theory that hypothesizes how investors behave rather than how investors should behave. Based on that hypothesized behavior of investors, a model that provides the expected return (a key input into constructing portfolios based on mean-variance analysis) is derived and is called an asset pricing model.

Together MPT and asset pricing theory provide a framework to specify and measure investment risk and to develop relationships between expected asset return and risk (and hence between risk and required return on an

## Exhibit 1
**The Legacy of MPT**

- **Asset allocation through mean-variance optimization**
- **Asset-liability management**
- **Bond portfolio immunization**
- **Optimal manager selection**
- **Value at risk (VaR)**
- **Tracking error budgeting**
- **Hedging strategies (e.g., currency overlay)**
- **Index funds/mutual funds**
- **Stable value/guaranteed investment contracts**
- **Factor models**
- **Long/short strategies**
- **Normal/balanced portfolios (e.g., life-cycle/lifestyle mutual funds)**
- **Funds of funds/managers of managers/ funds of hedge funds**

investment). However, it is critically important to understand that MPT is a theory that is independent of any theories about asset pricing. That is, the validity of MPT does not rest on the validity of asset pricing theory, a point that some critics of modern portfolio theory still appear not to understand.

This article begins with a succinct description of MPT followed by examples to illustrate the applications of modern portfolio theory. This section also presents a discussion on some of the issues associated with implementing the theory. We end with a brief conclusion.

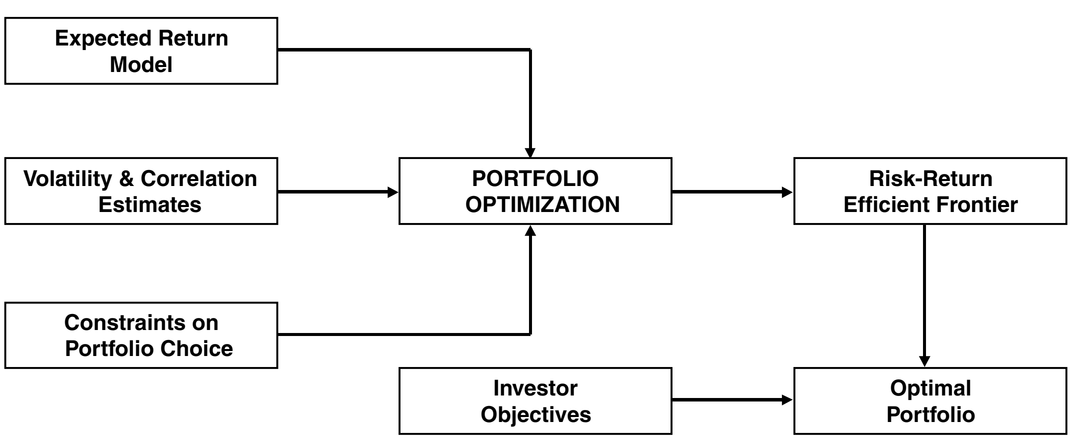
## DIVERSIFICATION AS A CENTRAL THEME IN FINANCE

Conventional wisdom has always dictated not putting all your eggs in one basket. In more technical terms, this adage is addressing the benefits of diversification. MPT quantified the concept of diversification, or "undiversification," by introducing the statistical notion of a covariance, or correlation. In essence, the adage means that putting all your money in investments that may all go broke at the same time, i.e., whose returns are highly correlated, is not a very prudent investment strategy—no matter how small the chance is that any one single investment will go broke. This is because if any one single investment goes broke, it is very likely due to its high correlation with the other investments, that the other investments are also going to go broke, leading to the entire portfolio going broke.

The concept of diversification is so intuitive and so strong that it has been continually applied to different areas within finance. Indeed, numerous innovations within finance have either been an application of the concept of diversification, or the introduction of new methods of obtaining improved estimates of the variances and covariances, thereby allowing for a more precise measure of diversification, and, consequently, for a more precise measure of risk. Exhibit 1 portrays the list of current applications that have directly or indirectly resulted as an outcome of MPT. This list is by no means all-inclusive.

While the examples in Exhibit 1 are applications of MPT to portfolio management and construction, the principles of diversification have been applied in other areas of finance. For example, the fastest-growing sector

## Exhibit 2
**The MPT Investment Process**



## EXHIBIT 3

**Annualized Returns Using Historical Performance Depend on Time Period** (%)

| Period | Lehman Aggregate | S&P 500 | MSCI EAFE | MSCI EM-Free |
|---|---|---|---|---|
| **Five Years** | | | | |
| 1991—1995 | 9.2 | 15.9 | 10.5 | 16.3 |
| 1996—2000 | 6.3 | 18.3 | 8.2 | 0.1 |
| **Ten Years** | | | | |
| 1991—2000 | 7.7 | 17.1 | 9.3 | 8.2 |

Source of monthly returns: Ibbotson Associates.

of the structured finance area is the collateralized debt obligation (CDO) market. A CDO is an asset-backed security backed by a pool of bonds or bank loans. In assigning a rating to tranches of a CDO, rating agencies have developed measures of the diversity of a portfolio in terms of industry concentration. The best-known such measure is Moody's diversity score.

### MEAN-VARIANCE OPTIMIZATION

Most readers are familiar with some model of asset allocation. Therefore, Exhibit 2 presents a summary of the MPT investment process (mean-variance optimization or the theory of portfolio selection).

Although the theory behind MPT is relatively straightforward, its implementation can get quite complicated. The theory dictates that given estimates of the returns, volatilities, and correlations of a set of investments and constraints on investment choices (for example, maximum exposures and turnover constraints), it is possible to perform an optimization that results in the risk/return or mean-variance efficient frontier.[2] This frontier is efficient because underlying every point on this frontier is a portfolio that results in the greatest possible expected return for that level of risk or results in the smallest possible risk for that level of expected return. The portfolios that lie on the frontier make up the set of efficient portfolios.

### MPT: A TOP-DOWN ASSET CLASS APPLICATION

One of the most direct and widely used applications of MPT is asset allocation. Because the asset allocation decision is so important, almost all asset managers and financial advisors determine an optimal portfolio for their clients—be they institutional or individual—by performing an asset allocation analysis using a set of asset classes.[3] They begin by selecting a set of asset classes (e.g., domestic large-cap and small-cap stocks, long-term bonds, international stocks). To obtain estimates of the returns and volatilities and correlations, they generally start with the historical performance of the indexes representing these asset classes.[4] These estimates are used as inputs in the mean-variance optimization which results in an efficient frontier. Then, using some criterion (for instance, using Monte Carlo simulations to compute the wealth distributions of the candidate portfolios), they pick an optimal portfolio. Finally, this portfolio is implemented using either index or actively managed funds.

### SOME THOUGHTS ON INPUTS BASED ON HISTORICAL PERFORMANCE

A number of approaches can be used to obtain estimates of the inputs that are used in a mean-variance optimization, and all approaches have their pros and cons. Since historical performance is the approach that is most commonly used, it may be helpful to present a discussion of this method. Exhibit 3 uses monthly returns over different time periods to present the annualized historical returns for four market indexes.

One drawback of using the historical performance to obtain estimates is clearly evident from this exhibit. Based on historical performance, a portfolio manager looking for estimates of the expected returns for these four asset classes to use as inputs for obtaining the set of efficient portfolios at the end of 1995 might have used the estimates from the five-year period 1991-1995. Then according to the portfolio manager's expectations, over the next five years, only the U.S. equity market (as represented by the S&P 500) outperformed, while U.S. bonds, Europe, Japan, and Emerging Markets all underperformed. In particular, the performance of Emerging Markets was

## EXHIBIT 4

**Annualized Standard Deviations Using Historical Performance Depend on Time Period** (%)

| Period | Lehman Aggregate | S&P 500 | MSCI EAFE | MSCI EM-Free |
|---|---|---|---|---|
| **Five Years** | | | | |
| 1991—1995 | 4.0 | 10.1 | 15.5 | 18.0 |
| 1996—2000 | 4.8 | 17.7 | 15.6 | 27.4 |
| **Ten Years** | | | | |
| 1991—2000 | 3.7 | 13.4 | 15.0 | 22.3 |

Source of monthly returns: Ibbotson Associates.

dramatically different from its expected performance (actual performance of 0.1% versus an expected performance of 16.3%). This finding is disturbing, because if portfolio managers cannot have faith in the inputs that are used to solve for the efficient portfolios, then it is not possible for them to have much faith in the outputs (i.e., the make-up and expected performance of the efficient and optimal portfolios).

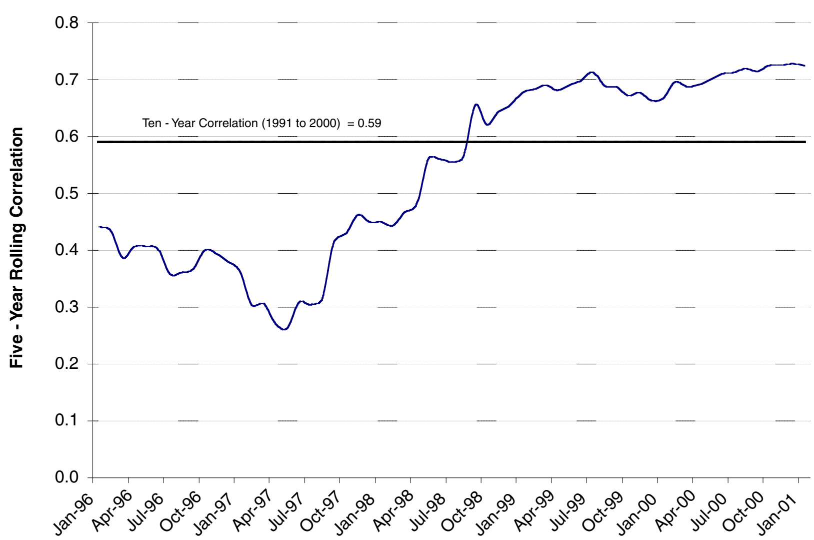
Portfolio managers who were performing the exercise at the beginning of 2001 faced a similar dilemma. Should they use the historical returns for the 1996-2000 period? That would generally imply that the optimal allocation has a large holding of U.S. equity (since that was the asset class that performed well), and an underweighting to U.S. bonds and emerging markets equity. But then what if the actual performance over the next five years is more like the 1991-1995 period? In that case the optimal portfolio is not going to perform as well as a portfolio that had a good exposure to bonds and emerging markets equity (note that emerging markets equity outperformed U.S. equity under that scenario). Or, should the portfolio managers use the estimates computed by using 10 years of monthly performance?

The truth is that there is no right answer because we are dealing with the world of uncertainty. This is also true for the cases of obtaining estimates for the variances and correlations. Exhibit 4 presents the standard deviations (square root of the variance) for the same indexes over the same time periods. Though the risk estimates for the Lehman Aggregate and EAFE indexes are quite stable, the estimates for the S&P 500 and EM-Free are significantly different over different time periods. However, the volatility of the indexes does shed some light on the problem of estimating expected returns as presented in Exhibit 4. MSCI EM- Free, the index with the largest volatility, also has the largest difference in the estimate of the expected return. Intuitively, this makes sense—the greater the volatility of an asset, the harder it is to predict its future performance.

Exhibit 5 shows the five-year rolling correlation between the S&P 500 and MSCI EAFE. In January 1996, the correlation between the returns of the S&P 500 and EAFE was about 0.45 over the prior five years (1991–1995). Consequently, a portfolio manager would have expected the correlation over the next five years to be around that estimate. However, for the five-year period ending December 2000, the correla-

## EXHIBIT 5

**Correlation Between Returns of S&P 500 and MSCI EAFE Indexes**



## Exhibit 6

**Histories Vary for Common Indexes for Asset Classes**

| Index | Asset Class | Inception Date |
|---|---|---|
| U.S. 30-Day T-Bill | U.S. Cash | 1/26 |
| Lehman Brothers Aggregate Bond | U.S. Bonds | 1/76 |
| S&P 500 | U.S. Large-Cap Equity | 1/26 |
| Russell 2000 | U.S. Small-Cap Equity | 1/79 |
| MSCI EAFE | Europe/Japan Equity | 1/70 |
| MSCI EM-Free | Emerging Markets Equity | 1/88 |

tion between the assets slowly increased to 0.73. Historically, this was an all-time high. In January 2001, should the portfolio manager assume a correlation of 0.45 or 0.73 between the S&P 500 and EAFE over the next five years? Or does 0.59, the correlation over the entire 10-year period (1991-2000), sound more reasonable?

Again, the truth is that there is no right answer. In reality, as mentioned earlier, if portfolio managers believe that the inputs based on the historical performance of an asset class are not a good reflection of the future expected performance of that asset class, they may objectively or subjectively alter the inputs. Different portfolio managers may have different beliefs, in which case the alterations will be different.[5] The important thing here is that all alterations have theoretical justifications, which, in turn, ultimately leads to an optimal portfolio that closely aligns to the future expectations of the portfolio manager.

There are some purely objective arguments as to why we can place more faith in the estimates obtained from historical data for some assets over others. Exhibit 6 presents the commonly used indexes for some asset classes and their respective inception dates. Since there are varying lengths of histories available for different assets (for instance, U.S. and European markets not only have longer histories, but their data are also more accurate), inputs of some assets can generally be estimated more precisely than the estimates of others.[6]

When solving for the efficient portfolios, the differences in precision of the estimates should be explicitly incorporated into the analysis. But MPT assumes that all estimates are as precise or imprecise, and therefore treats all assets equally. Most commonly, practitioners of mean-variance optimization incorporate their beliefs on the precision of the estimates by imposing constraints on the maximum exposure of some asset classes in a portfolio. The asset classes on which these constraints are imposed are generally those whose expected performances are either harder to estimate, or those whose performances are estimated less precisely.[7]

The extent to which we can use personal judgment to subjectively alter estimates obtained from historical data depends on our understanding what factors influence the returns on assets, and what is their impact. The political environment within and across countries, monetary and fiscal policies, consumer confidence, and the business cycles of sectors and regions are some of the key factors that can assist in forming future expectations of the performance of asset classes.

To summarize, it would be fair to say that using historical returns to estimate parameters that can be used as inputs to obtain the set of efficient portfolios depends on whether the underlying economies giving rise to the observed outcomes of returns are strong and stable. Strength and stability of economies comes from political stability and consistency in economic policies. It is only after an economy has a lengthy and proven record of

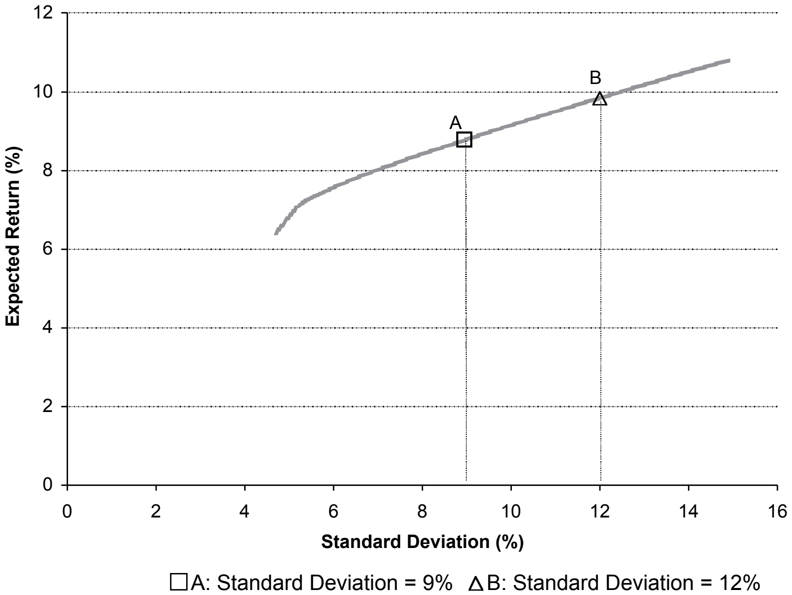
## Exhibit 7

**Forward-Looking Inputs (Expected Returns, Standard Deviations, and Correlations)**

| E(R) | SD(R) | Asset Classes | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| 6.4% | 4.7% | U.S. Bonds | **1** | **1.00** | | | |
| 10.8 | 14.9 | U.S. Large-Cap Equity | **2** | 0.32 | **1.00** | | |
| 11.9 | 19.6 | U.S. Small-Cap Equity | **3** | 0.06 | 0.76 | **1.00** | |
| 11.5 | 17.2 | EAFE International Equity | **4** | 0.17 | 0.44 | 0.38 | **1.00** |

## EXHIBIT 8

**Efficient Frontier Using Only U.S. Bonds and U.S. Large-Cap Equity**




healthy and consistent performance under varying (political and economic) forces that impact free markets that historical performance of its markets can be seen as a fair indicator of their future performance.

### PORTFOLIO SELECTION: AN EXAMPLE

Using an explicit example, we now illustrate how asset managers and financial advisors use MPT to build optimal portfolios for their clients. In this example we will construct an efficient frontier made up of U.S. bonds and U.S. and international equity, and shed some light on the selection of an optimal portfolio. Exhibit 7 presents the forward-looking assumptions for the four asset classes.

These inputs are an example of estimates that are *not* totally based on historical performance of these asset classes. The expected return estimates are created using a risk premium approach (i.e., obtaining the historical risk premiums attached to bonds, large-cap, mid-cap, small-cap, and international equity) and then have been subjectively altered to include the asset manager's expectations regarding the future long-run (5 to 10 years) performance of these asset classes. The risk and correlation figures are mainly historical.

The next step is to use a software package to perform the optimization that results in the efficient frontier. For purposes of exposition, Exhibit 8 presents the efficient frontier using only two of the four asset classes from Exhibit 7—U.S. bonds and large-cap equity. We highlight two efficient portfolios on the frontier: A and B, corresponding to standard deviations of 9% and 12%, respectively. Portfolio B has the higher risk, but it also has the higher expected return. We suppose that one of these two portfolios is the optimal portfolio for a hypothetical client.

Exhibit 9 presents the compositions of portfolios A and B, and some important characteristics that may assist in

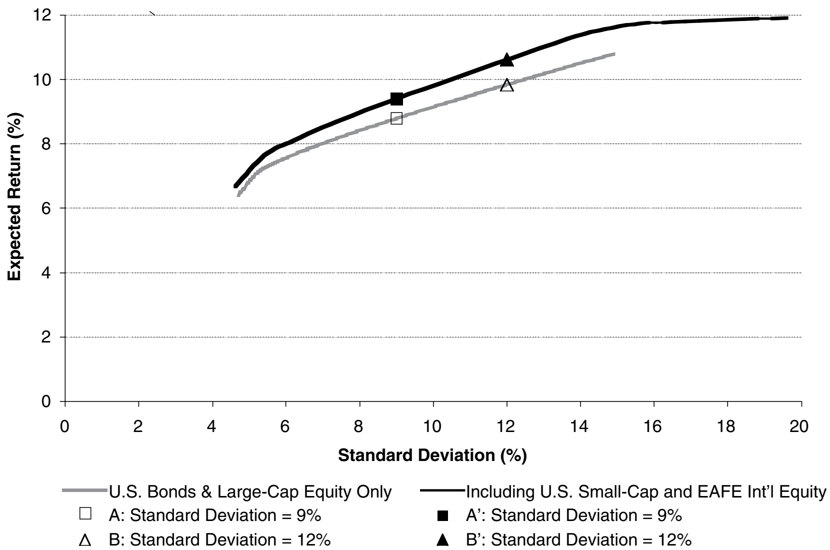
## EXHIBIT 9

**Growth of $100—Monte Carlo Wealth Distributions Illustrate Risk/Return Trade-Off of Portfolios A and B**

| **Characteristic** | **Portfolio A** | | | **Portfolio B** | | |
|---|---|---|---|---|---|---|
| U.S. Fixed-Income Allocation | 45.80% | | | 22.00% | | |
| U.S. Large-Cap Equity Allocation | 54.20 | | | 78.00 | | |
| | | | | | | |
| Expected Return | 8.79% | | | 9.83% | | |
| Standard Deviation | 9.00% | | | 12.00% | | |
| Return per Unit of Risk | 98 basis points (bp) | | | 82 basis points (bp) | | |
| | | | | | | |
| **Growth of $100** | 1 Year | 5 Years | 10 Years | 1 Year | 5 Years | 10 Years |
| 95th Percentile (Upside) | $124 | $203 | $345 | $131 | $232 | $424 |
| Average (Expected) | 109 | 152 | 232 | 110 | 160 | 255 |
| 5th Percentile (Downside) | 95 | 111 | 146 | 91 | 104 | 137 |

Note: Assumes annual rebalancing.

**E X H I B I T  10**
**Expanding Efficient Frontier Using All Asset Classes**




the selection of the optimal portfolio for the client. As one would expect, the more conservative portfolio (A) allocates more to the conservative asset class. Portfolio A allocated a little more than 45% of the portfolio to fixed income, while portfolio B allocates only 22% to that asset class. This results in significantly higher standard deviation for Portfolio B (12% versus 9%). In exchange for the 3% (or 300 basis points) of higher risk, portfolio B results in 104 basis points of higher expected return (9.83% versus 8.79%). This is the risk/return trade-off that the client faces. Does the increase in the expected return compensate the client for the increased risk that she will be bearing?

As mentioned earlier, another approach to selecting between the efficient portfolios is to translate the differences in risk in terms of differences in the wealth distribution over time. The higher the risk, the wider the spread of the distribution. A wider spread implies a greater upside and a greater downside. Exhibit 9 also presents the 95th percentile, expected, and 5th percentiles for $100 invested in portfolios A and B over 1, 5, and 10 years, respectively.[8] Over a one-year period, there is a 1-in-20 chance that the $100 invested in portfolio A will grow to $124, but there is also a 1-in-20 chance that the portfolio will lose $5 (i.e., it will it shrink to $95). In comparison, for portfolio B there is a 1-in-20 chance that $100 will grow to $131 (the upside is $6 more than if invested in portfolio A). But there is also a 1-in-20 chance that the portfolio will shrink to $91 (the downside is $4 more than if invested in portfolio A). If the investment horizon is one year, is this investor willing to accept a 1-in-20 chance of losing $9 instead of $4 for a 1-in-20 chance of gaining $31 instead of $24?[9] The answer depends on the investor's risk aversion.

As the investment horizon becomes longer, the chances that a portfolio will lose its principal keep declining. Over 10 years, there is a 1-in-20 chance that portfolio A will grow to $345, but there is also a 1-in-20 chance that the portfolio will only grow to $146 (the chances that the portfolio results in a balance less than $100 are much smaller). In comparison, over 10 years, there is a 1-in-20 chance that portfolio B will grow to $424 (the upside is $79 more than if invested in portfolio A)! And even though there is a 1-in-20 chance that the portfolio will only grow to $137—that is, only $9 less than if invested in portfolio A! Also, portfolio B's average (expected) balance over 10 years is $23 more than portfolio A's ($255 versus $232). Somehow, compounding makes the more risky portfolio seem more attractive over the longer run. In other words, a portfolio that may not be acceptable to the investor over a short run may be acceptable over a longer investment horizon. In summary, it is sufficient to say that the optimal portfolio depends not only on risk aversion, but also on the investment horizon.

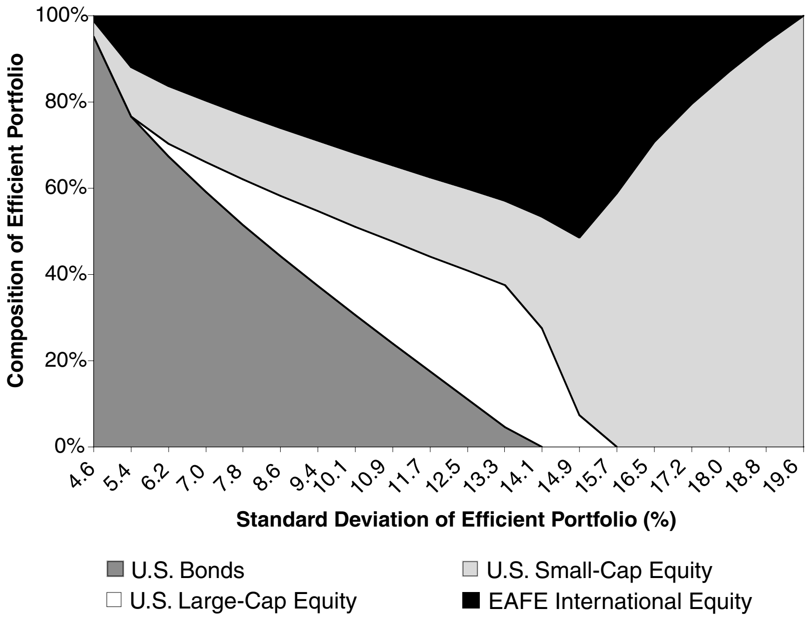
## INCLUSION OF MORE ASSET CLASSES

Exhibit 10 compares the efficient frontier using two asset classes, namely, U.S. bonds and large-cap equity, with one obtained from using all four asset classes in the optimization. The inclusion of U.S. small-cap and EAFE international equity into the mix makes the opportunity set bigger (i.e., the frontier covers a larger risk/return spectrum). It also moves the efficient frontier outward (i.e., the frontier results in a larger expected return at any given level of risk, or, conversely, results in a lower risk for any given level of expected return). The frontier also highlights portfolios A′ and B′—the portfolios with the same standard deviation as portfolios A and B, respectively.

Exhibit 11 shows the composition of the underlying portfolios that make up the frontier. Interestingly, U.S. small-cap and EAFE international equity—the more aggressive asset classes—are included in all the portfolios.

## EXHIBIT 11

**Composition of Efficient Frontier**




equity results in sizable increases in the expected return and return per unit of risk. In particular, the conservative portfolio A′ has an expected return of 9.39% (60 basis points over portfolio A) and the aggressive portfolio B′ has an expected return of 10.61% (78 basis points over portfolio B). Note also that there is an increase in the returns per unit of risk.

The huge allocations to U.S. small-cap and EAFE international equity in portfolios A′ and B′ may make some investors uncomfortable. U.S. small-cap equity is the most risky asset class and EAFE international equity is the second most aggressive asset class. The conservative portfolio allocates more than 40% of the portfolio to these two asset classes, while the aggressive allocates more than 50%. As discussed in the section on using inputs based on historical returns, these two would also be the asset classes whose expected returns would be harder to estimate. Consequently, investors may not want to allocate more than a certain amount to these two asset classes.

On a separate note, investors in the U.S. may also want to limit their exposure to EAFE international equity. This may be simply for psychological reasons. Familiarity leads them to believe that domestic asset classes are less risky.[10] Exhibit 13 presents the composition of the efficient frontier when the maximum allocation to EAFE is constrained at 10% of the portfolio. As a result of this constraint, all the portfolios now receive an allocation of U.S. large-cap equity.

Exhibit 14 compares the composition of portfolios A′ and B′ to that of portfolios A″ and B″, the respective

Even the least risky portfolio has a small allocation to these two asset classes. On the other hand, U.S. large-cap equity—an asset class that is thought of as the backbone of a domestic portfolio—gets excluded from the more aggressive portfolios.

Exhibit 12 compares the composition and expected performance of portfolios A and B to A′ and B′, respectively. Both the new portfolios A′ and B′ find U.S. small-cap and EAFE international equity very attractive and replace a significant proportion of U.S. large-cap equity with those asset classes. In portfolio B′, the more aggressive mix, the allocation to U.S. bonds also declines (15.1% versus 22%).

Inclusion of U.S. small-cap and EAFE international

## EXHIBIT 12

**Composition of Equally Risky Efficient Portfolios on Expanded Frontier**

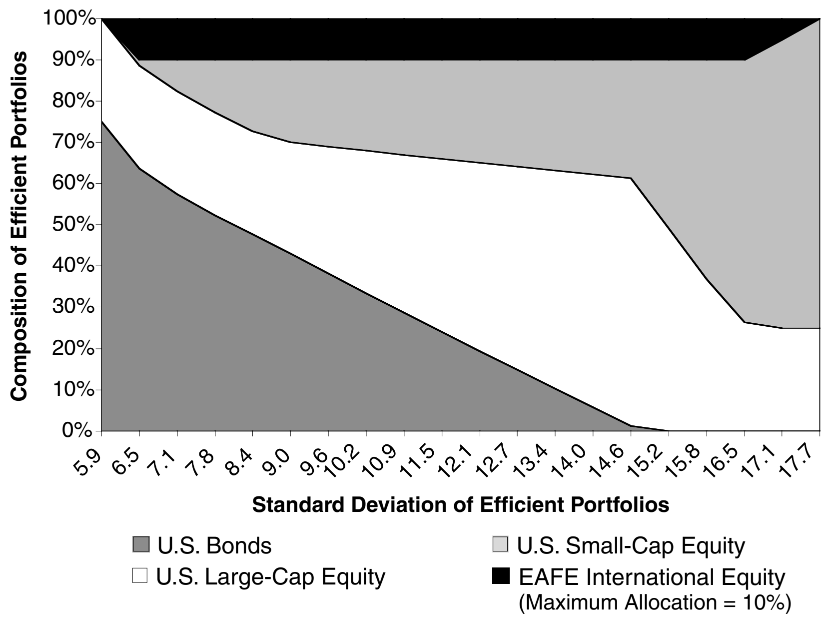
| Asset Class | Standard Deviation = 9.0% | | Standard Deviation = 12.0% | |
|---|---|---|---|---|
| | A | A′ | B | B′ |
| U.S. Fixed-Income | 34.30% | 40.40% | 22.00% | 15.10% |
| U.S. Large-Cap Equity | 18.70 | 15.80 | 78.00 | 27.80 |
| U.S. Small-Cap Equity | - | 16.10 | - | 18.60 |
| EAFE International Equity | - | 27.70 | - | 38.50 |
| | | | | |
| Expected Return | 8.79% | 9.39% | 9.83% | 10.61% |
| Standard Deviation | 9.00% | 9.00% | 12.00% | 12.00% |
| Return per Unit of Risk | 98 bps | 104 bps | 82 bps | 88 bps |

equally risky portfolios that lie on the constrained efficient frontier. In the conservative portfolio A″, the combined allocation to U.S. small-cap and EAFE international equity has declined to 30% (from 43.8%), and in B″ it has fallen to 34.8% (from 57.1%). Also, now the bond allocation increases for both portfolios.

The decline in the expected return can be used to quantify the cost of this constraint. The conservative portfolio's expected return fell from 9.39% to 9.20%—a decline of 19 basis points. This cost may be well worth it for an investor whose optimal appetite for risk is 9%. The more aggressive portfolio pays more for the constraint (10.61% – 10.26% = 35 basis points).[11]

**EXHIBIT 13**

**Composition of Constrained Efficient Frontier—Maximum Allocation to EAFE International Equity = 10%**




## EXTENSIONS OF THE BASIC ASSET ALLOCATION MODEL

In mean-variance analysis, the variance (standard deviation) of returns is the proxy measure for portfolio risk. As a supplement, the probability of not achieving a portfolio expected return can also be calculated. This type of analysis, referred to as *risk-of-loss analysis*, would be useful in determining the most appropriate mix from the set of optimal portfolio allocations.[12] In the context of setting investment strategy for a pension fund that has a long-term normal asset allocation policy established, the value of the probability of loss for the desired return benchmark over the long-term horizon can be used as the maximum value for the short term. For example, if the long-term policy has a 15% probability of loss for 0% return, the mix may be changed over the short run, as long as the probability of loss of the new mix has a maximum of 15%. Therefore, by taking advantage of short-term expectations to maximize return, the integrity of the long-term policy is retained. A floor or base probability of loss is therefore established that can provide boundaries within which strategic return/risk decisions may be made. As long as the alteration of the asset allocation mix does not violate the probability of

**EXHIBIT 14**

**Benefits and Costs of Constraining an Efficient Frontier**

| Asset Class | Unconstrained | | Maximum Allocation to EAFE International Equity = 10.0% | |
|---|---|---|---|---|
| | A′ | B′ | A″ | B″ |
| U.S. Fixed-Income | 40.40% | 15.10% | 43.10% | 20.10% |
| U.S. Large-Cap Equity | 15.80 | 27.80 | 26.90 | 45.10 |
| U.S. Small-Cap Equity | 16.10 | 18.60 | 20.00 | 24.80 |
| EAFE International Equity | 27.70 | 38.50 | 10.00 | 10.00 |
| Expected Return | 9.39% | 10.61% | 9.20% | 10.26% |
| Standard Deviation | 9.00% | 12.00% | 9.00% | 12.00% |
| Cost of Constraint | - | - | 19 bp | 35 bp |

Note: Assumes annual rebalancing.

## EXHIBIT 15
**FR 100 Growth and FR 1000 Value Performance**

| | 20 Years Ending 2001 | |
|---|---|---|
| | FR 1000 Growth | FR 1000 Value |
| Annualized Return | 14.3% | 16.1% |
| Standard Deviation | 19.9% | 14.0% |
| Correlation | 0.80 | |

Data from Ibbotson Associates.

loss, increased return through strategies such as tactical asset allocation can be pursued.

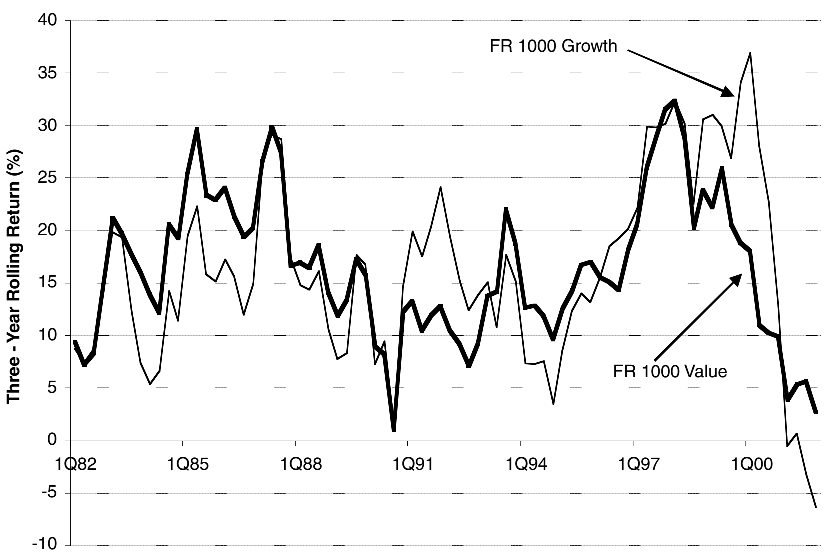
Mean-variance analysis has been extended to multiple possible scenarios. Each assumed scenario is believed to be an assessment of the asset performance in the long run, over the investment horizon. A probability can be assigned to each scenario so that an efficient set can be constructed for the composite scenario. It is often the case, however, that an investor expects a very different set of input values in mean-variance analysis that are applicable in the short run, say, the next 12 months. For example, the long-term expected return on equities may be estimated at 15%, but over the next year the expected return on equities may be only 5%. The investment objectives are still stated in terms of the portfolio performance over the entire investment horizon. However, the return characteristics of each asset class are described by one set of values over a short period and another set of values over the balance of the investment horizon. A mean-variance analysis can be formulated that simultaneously optimizes over the two periods.[13]

Finally, mean-variance analysis has been extended to explicitly incorporate the liabilities of pension funds.[14] This extension requires not only the return distribution of asset classes that must be considered in an optimization model, but also the liabilities.

## CURRENT VARIATIONS ON THE MPT THEME

The basic asset allocation model has been applied to many areas within finance. In this section we briefly describe four major applications that are derived from MPT: asset allocation implementation, factor models and portfolio construction, risk management by sell-side firms, and managing active risk.

## EXHIBIT 16
**Three-Year Rolling Returns of FR 1000 Growth and FR 1000 Value**




### Implementing Asset Allocation by Diversifying Using Different Styles

An application of MPT commonly used by financial planners is in the implementation of their client's asset allocation. We saw in the previous section how MPT is applied in a top-down fashion to come up with an optimal portfolio using the expected performance of the asset classes and the risk appetite of the client. However, instead of directly investing in indexes that represent the asset class, implementation is performed using managers with different styles that make up the asset class.[15]

Exhibit 15 presents the performance of the growth and value indexes that make up the FR 1000 Index—one benchmark for the domestic large-cap equity universe. Over the 20 years ending 2001, the FR

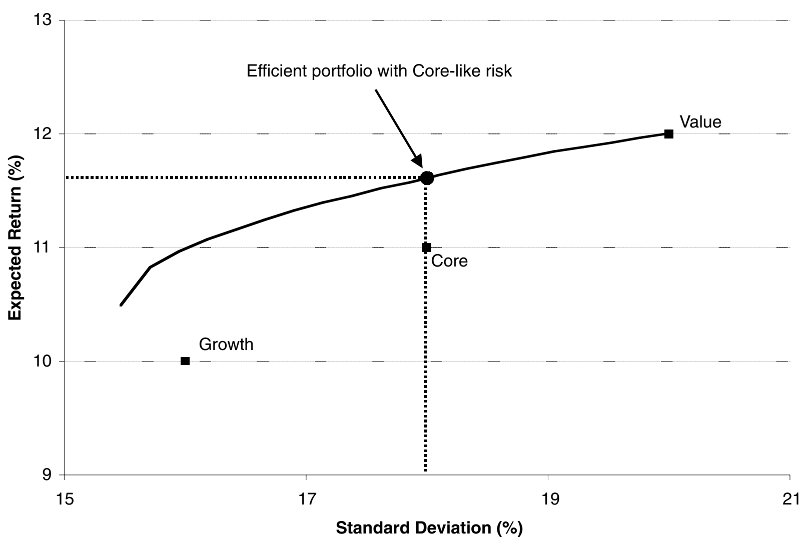
## EXHIBIT 17

**Expected Performance of Large-Cap Equity Styles** (Hypothetical)

| Expected Return | Standard Deviation | Large-Cap Equity Styles | Correlations V | C | G |
|---|---|---|---|---|---|
| 12% | 20% | Value (V) | 1.00 | | |
| 11% | 18% | Core (C) | 0.75 | 1.00 | |
| 10% | 16% | Growth (G) | 0.60 | 0.75 | 1.00 |

## EXHIBIT 18

**Efficient Portfolio of Large-Cap Equity Allocates to All Styles**




1000 value has outperformed the FR 1000 growth and has also resulted in lower risk.

Exhibit 16 shows that the short-run performance between the two indexes varies dramatically. In the 1990s growth outperformed value by a significant margin; more recently, value has begun to outperform growth. Consequently, a financial advisor may believe that over the next three to five years value is going to outperform growth. Exhibit 17 presents a hypothetical set of beliefs for such a financial planner.

Exhibit 18 shows the efficient frontier made up of the three styles. Notice that neither the growth index nor the core index lies on the frontier, meaning that given the beliefs for the expected performance in Exhibit 17, a strategy that employs a 100% allocation to the core index (to obtain exposure to the domestic large-cap universe) would be inefficient.

Exhibit 19 presents the optimal portfolio for a client to obtain core-like risk. Under this scenario, 69% of the portfolio is allocated to a value manager, 23% to a core manager, and 8% to a growth manager. This combination results in 60 basis points of expected return over a portfolio that allocates 100% to a core style.

### Factor Models and Portfolio Construction

Application of mean-variance analysis for portfolio construction requires a significantly greater number of inputs to be estimated—expected return for each security, variance of returns for each security, and either covariance or correction of returns between each pair of securities. For example, a mean-variance analysis that allows 200 securities as possible candidates for portfolio selection requires 200 expected returns, 200 variances of return, and 19,900

## EXHIBIT 19

**Optimal Allocation to Styles for Core-Like Risk**

| Large-Cap Equity Styles | without Style Optimization | with Style Optimization |
|---|---|---|
| Value (V) | - | 69.0% |
| Core (C) | 100.0% | 23.0% |
| Growth (G) | - | 8.0% |
| | | |
| Expected Return | 11.0% | 11.6% |
| Standard Deviation | 18.0% | 18.0% |

correlations or covariances. An investment team tracking 200 securities may reasonably be expected to summarize its analysis in terms of 200 means and variances, but it is clearly unreasonable for it to produce 19,900 carefully considered correlation coefficients or covariances.

It was clear to Markowitz [1959] that some kind of model of covariance structure was needed for the practical application of normative analysis to large portfolios. He did little more than point out the problem and suggest some possible models of covariance for research. One model Markowitz proposed to explain the correlation structure among security returns assumed that the return on the *i*-th security depends on an "underlying factor, the general prosperity of the market as expressed by some index" [p. 100]. Mathematically, the relationship is expressed as follows:

$$R_i = \alpha_i + \beta_i F + u_i$$

where $R_i$ = the return on security *i;*
$F$ = value of some index;[16] and
$u_i$ = error term

The expected value of $u_i$ is zero and $u_i$ is uncorrelated with $F$ and every other $u_j$. Markowitz further suggested that the relationship need not be linear and that there could be several underlying factors.

In 1963, Sharpe used the above equation as an explanation of how security returns tend to go up and down together with a general market index, *F*. He called the model given by the above equation the *market model*.[17] He concluded that the market model was as complex a covariance as seemed to be needed. This conclusion was supported by Cohen and Pogue [1967]. However, King [1966] found strong evidence for industry factors in addition to the marketwide factor. Rosenberg [1974] found other sources of risk beyond marketwide factor and industry factor.

The arbitrage pricing model formulated by Ross [1976] provides theoretical support for an asset pricing model where there is more than one risk factor. Academics and commercial vendors have developed multifactor risk models that can be used in the construction and risk control of a portfolio instead of the full mean-variance approach. These models fall into three categories: statistical factor models, macroeconomic factor models, and fundamental factor models.[18] The most commonly used model for the construction of efficient portfolios is the fundamental factor model. There are several fundamental factor models available from commercial vendors; the most popular one among institutional investors and consultants to pension funds is the Barra fundamental factor model.

The basic relationship to be estimated in a multifactor risk model is

$$R_i - R_f = \beta_{i,F1} R_{F1} + \beta_{i,F2} R_{F2} + \ldots + \beta_{i,FH} R_{FH} + e_i$$

where

$R_i$ = rate of return on stock *i*;
$R_f$ = risk-free rate of return;
$\beta_{i,Fj}$ = sensitivity of stock *i* to risk factor *j;*
$R_{Fj}$ = rate of return on risk factor *j*; and
$e_i$ = nonfactor (specific) return on security *i*.

Fundamental factor models use company and industry attributes and market data as "descriptors." Examples are price/earnings ratios, book/price ratios, estimated earnings growth, and trading activity. The estimation of a fundamental factor model begins with an analysis of historical stock returns and descriptors about a company. In the Barra model, for example, the process of identifying the risk factors begins with monthly returns for 1,900 companies that the descriptors must explain. Descriptors are not the risk factors but instead are the candidates for risk factors. The descriptors are selected in terms of their ability to explain stock returns. That is, all of the descriptors are potential risk factors, but only those that appear to be important in explaining stock returns are used in constructing risk factors. Once the descriptors that are statistically significant in explaining stock returns are identified, they are grouped into "risk indices" to capture related company attributes. For example, descriptors such as market leverage, book leverage, debt-to-equity ratio, and company's debt rating are combined to obtain a risk index referred to as "leverage." Thus, a risk index is a combination of descriptors that captures a particular attribute of a company.

The construction of efficient portfolios is then formulated in terms of the risk factors rather than the full mean-variance analysis.[19] Typically, the application is the construction of a portfolio where the benchmark is a market index. Rather than variance of return being the measure of risk, it is the tracking error (the standard deviation of the difference between the return on the portfolio and the return on the benchmark index) that is the measure of risk exposure.

## EXHIBIT 20

**Hypothetical Alpha-Tracking Error Performance of Large-Cap Active Managers**

| Expected Alpha | Tracking Error | Large-Cap Managers | Alpha Correlations I | E | C | A |
|---|---|---|---|---|---|---|
| 0.00% | 0.00% | Indexed (I) | 1.00 | | | |
| 1.00% | 1.75% | Enhanced (E) | 0.10 | 1.00 | | |
| 1.50% | 3.00% | Core (C) | –0.10 | 0.10 | 1.00 | |
| 2.00% | 4.50% | Active (A) | –0.25 | –0.10 | 0.10 | 1.00 |

### VaR Measurement and Management for Dealer Positions

The now widely used value-at-risk framework (VaR) for the measurement and management of market risk for financial markets is based on the concepts first formalized in MPT.[20] The need to consider each security or financial instrument in the context of the overall exposure and not in isolation was the key to obtaining more precise estimates of the day-to-day risks faced by a financial institution, and thereby allowing the institution to keep the VaR within tolerable levels.

An example may assist in clarifying the impact of correlations on the day-to-day VaR of a financial institution. If a U.S.-based investor holds a position in a euro-denominated bond, then the investor has exposure to two risk factors: 1) interest rate risk that can directly impact the value of the bond and 2) foreign exchange risk (i.e., the volatility of the euro/USD exchange rate). But when computing the risk of this position, it is important to keep in mind that the total risk of this position is not simply the sum of the interest rate risk and the foreign-exchange risk, but rather must incorporate the impact of the correlation that exists between the returns on the euro-denominated bond (i.e., the interest rate risk) and the euro/USD exchange rate (i.e., foreign exchange risk). Extensive work and research has been done so as to collect more accurate data on the performance of a vast array of financial instruments and to improve the methods used to compute the estimates of the variances and covariances.[21]

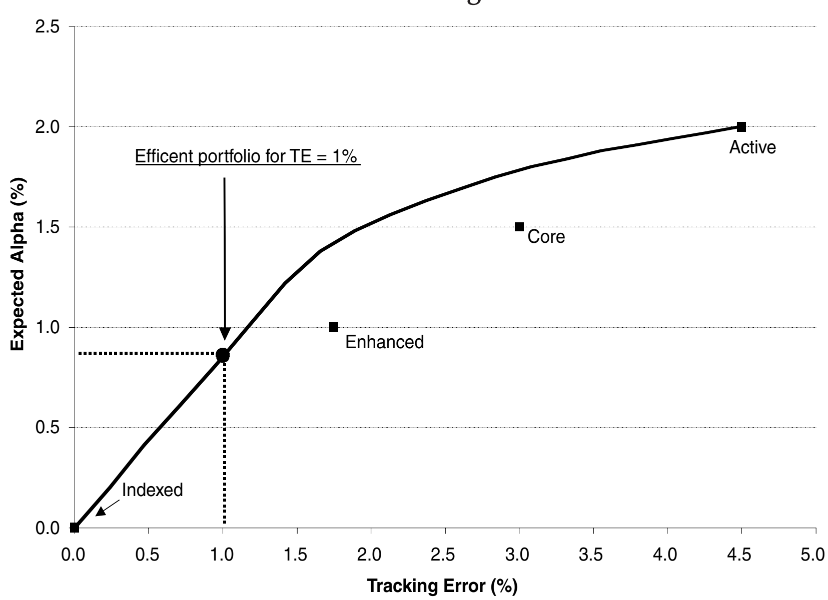
### Managing Active Risk or Tracking Error Budgeting

For some time now, institutional investors have been working with asset managers in the pursuit of a methodology to manage the active risk associated with their portfolios relative to a benchmark (i.e., tracking risk). The methodology, they hope, will permit them to budget, or allocate, risk across their active managers. Also, if the methodology is transparent, so that ex ante risk/return expectations can be formed, then it is possible to reward investors with the maximum return for the level of risk undertaken. In other words, it is possible to efficiently allocate the active risk across managers, thereby making the active allocation decision efficient.

The key to constructing such a methodology lies in the implicit relationship between the performance of active managers and the deviations from their portfolios'

## EXHIBIT 21

**Efficient Allocation to All Active Managers Diversifies Active Risk**



## EXHIBIT 22

**Hypothetical Domestic Large-Cap Manager Mix for Tracking Error = 1%**

| Large-Cap Managers | Efficient Allocation for TE = 1% |
|---|---|
| Indexed (I) | 34.0% |
| Enhanced (E) | 38.0% |
| Core (C) | 16.0% |
| Active (A) | 12.0% |
| | |
| Alpha | 86 bps |
| Tracking Error | 100 bps |
| Information Ratio | 0.86 |

benchmark. Just as the key to constructing the set of optimal portfolios lies in forming expectations of the risk/return characteristics of the various asset classes, understanding the alpha/tracking error relationships across different managers is the first step in creating a "tracking error budgeting" methodology.[22]

Exhibit 20 presents the hypothetical alpha/tracking error relationships for four types of domestic large-cap equity managers. The exhibit also presents the correlations among the alphas for the different managers. The indexed manager is not expected to beat the benchmark and is also expected to track the benchmark perfectly. Consequently, this type of manager has an expected alpha of zero with no tracking error. An enhanced indexed or risk-controlled manager may be expected to generate an alpha of 1% with a relatively low tracking error of 1.75%. A risk-controlled manager seeks to produce alpha by making bets only along one dimension: style, size, or sector. A core manager could make bets along more than one of those dimensions, thereby seeking to generate a larger alpha of 1.5% with a larger tracking error of 3%. Finally, a more active manager may choose to perform some stock selection, and so would have the largest active risk—a tracking error of 4.5%—but would also be expected to generate the largest alpha.

The correlations are generally manager-specific, but if manager styles are complementary (i.e., if managers are not seeking alpha from the same sources), then the manager alphas will most likely be uncorrelated or even negatively correlated. As expected, the most benefits from active risk diversification will come from these types of managers.

Exhibit 21 shows the efficient frontier in tracking error/alpha space for this set of managers. The efficient portfolios change along the efficient frontier, going from a 100% allocation to the indexed manager (for no alpha and tracking error) to 100% allocated to the active manager (for an alpha of 2% and a tracking error of 4.5%).

Exhibit 22 shows the optimal portfolio for a tracking error budget of 100 basis points. The reason for restricting the active risk budget to such a small level is that for domestic large-cap equity the return/risk ratio falls after a tracking error of about 1.5% (see the slope of the efficient frontier). Therefore, after that level the investor receives a smaller alpha reward for each unit of tracking error.

## CONCLUSION

By now it is evident that MPT, the theory first expounded by Markowitz 50 years ago, has found applications in many aspects of modern financial theory and practice. We have illustrated a few of the most widely used applications in the areas of asset allocation, portfolio management, and portfolio construction. Though it did take a few years to create a buzz, the late 20th and early 21st centuries saw no let-up in the spread of the application of MPT. Further, it is unlikely that its popularity will wane anytime in the near or distant future. Consequently, it seems safe to predict that MPT will occupy a permanent place in the theory and practice of finance.

## ENDNOTES

[1]This is partly also a result of advances in technology.

[2]In practice, this optimization is performed using an off-the-shelf asset allocation package.

[3]Brinson, Hood, and Beebower [1986, 1991] provide evidence that leads them to conclude that asset allocation is a major determinant of portfolio performance.

[4]Not all institutional asset managers use this method to obtain estimates of expected returns.

[5]It is quite common that the optimal strategic bond/equity mix within a portfolio differs significantly across portfolio managers.

[6]Statistically, the precision of an estimate is directly proportional to the amount of information used to estimate it. That is, the more data used to obtain an estimate, the greater the precision of the estimate.

[7]An alternative method for incorporating beliefs into MPT is presented in Black and Litterman [1991].

[8]The 95th percentile captures the upside associated with a 1-in-20 chance, while the 5th percentile represents the downside associated with a 1-in-20 chance.

[9]It may be useful to mention here that more recently researchers in behavioral finance have found some evidence to suggest that investors view the upside and downside differently. In particular, they equate each downside dollar to more than one upside dollar. For a good review of the behavioral finance literature, see Shefrin [2001].

[10]Similarly, investors in Europe may believe that EAFE equity is less risky than U.S. equity and may want to limit their exposure to U.S. asset classes.

[11]For a discussion of the benefits and costs of constraints, see Gupta and Eichhorn [1998].

[12]Risk-of-loss analysis, as well as the multiple scenario analysis and short-term/long-term analysis described next, were developed by Gifford Fong Associates in the early 1980s. Descriptions are provided in Fong and Fabozzi [1985].

[13]For more on this, see Markowitz and Perold [1981].

[14]See Leibowitz, Kogelman, and Bader [1992]. The mean-variance model they present strikes a balance between 1) asset performance and the maintenance of acceptable levels of its downside risk and 2) surplus performance and the maintenance of acceptable levels of its downside risk.

[15]Sharpe [1992] advocates analyzing manager styles. Consultants popularized Sharpe's methodology by characterizing managers according to style boxes. While this was not Sharpe's original intent, this is perhaps one reason this application of MPT is so common.

[16]Markowitz [1959] uses the notation *I* in proposing the model.

[17]Notice that a parameter to be estimated in the equation is beta. In the Sharpe [1964] formulation of the capital asset pricing model, a proxy for systematic market risk is also labeled beta. The betas in the market model and the CAPM are not the same constructs. It is important to differentiate these two beta measures, and failure to do so has led to confusion. for example, in 1980, *Institutional Investor* published an article with the title "Is Beta Dead?" (Wallace [1980]).

Markowitz [1984] has explained that the major reason for the debate is the confusion between the beta associated with the market model (estimated to avoid having to compute all covariances for assets in a portfolio, and the index need not be mean-variance efficient) and the beta in the CAPM, which uses a market portfolio that should be mean-variance efficient.

[18]See Connor [1995] for a review of each type of model.

[19]For an illustration of using a fundamental factor model to construct a portfolio and control its risk for equity portfolios, see Fabozzi, Jones, and Vardharaj [2002]. The application of a multifactor risk model for the construction of a fixed-income portfolio is provided by Dynkin and Hyman [2002].

[20]VaR is the maximum value that a portfolio may lose over a given time period with a given level of confidence. A one-day period and a 95% VaR of $1 million means there is a 5% chance that the portfolio can lose $1 million over the next day.

[21]For a more detailed description of VaR, see "RiskMetrics—Technical Documentation" [1996].

[22]For research that addresses the alpha-tracking error relationships across asset classes, see Gupta, Prajogi, and Stubbs [1999].

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